AD-A084 794   FLORIDA STATE UNIV TALLAHASSEE DEPT OF STATISTICS   F/G 12/1
A MULTIVARIATE NEW BETTER THAN USED CLASS DERIVED FROM A SHOCK --ETC(U)
MAR 80 E EL-NEWEIHI, F PROSCHAN, J SETHURAMAN DAAG29-79-C-0158
UNCLASSIFIED   ARO-16713.9-M   NL

1 OF 1
AD
A084794

END
DATE
FILMED
6-80
DTIC

A Multivariate New Better Than Used Class Derived from a Shock Model

by

Emad El-Neweihi[1], Frank Proschan[2], and Jayaram Sethuraman

FSU Statistics Report M540
AFOSR Technical Report No. 78-105

The Florida State University
Department
of
Statistics
Tallahassee, Florida
32306

A Multivariate New Better Than Used Class
Derived from a Shock Model

by

Emad El-Neweihi[1], Frank Proschan[2], and Jayaram Sethuraman[3]

FSU Statistics Report M540
AFOSR Technical Report No. 78-105
USARO Technical Report No. D45

March, 1980
The Florida State University
Department of Statistics
Tallahassee, Florida 32306

[1]Research sponsored by the Air Force Office of Scientific Research under Grant AFOSR 76-3050C. Affiliation: Department of Mathematics, University of Illinois at Chicago Circle.

[2]Research sponsored by the Air Force Office of Scientific Research under Grant AFOSR 78-3678.

[3]Research sponsored by the U.S. Army Research Office under Grant DAAG-29-79-C-D158.

AMS 1970 Subject Classifications: Primary 60K10; Secondary 62 N05.

Key words and phrases: New better than used, multivariate new better than used, shock model, multivariate exponential, reliability, life distribution, survival function.

Abstract.

We introduce a new class of multivariate new better than used (MNBU) life distributions based on a shock model similar to that yielding the Marshall-Olkin multivariate exponential distribution. Let $T_1, \ldots, T_M$ be independent new better than used (NBU) life lengths. Let $\overline{F}(t_1, \ldots, t_n)$ be the joint survival function of $\min_{j \in A_i} T_j$, $i = 1, \ldots, n$, where $A_1, \ldots, A_n$ are nonempty subsets of $\{1, \ldots, M\}$ and $\bigcap_{i=1}^{n} A_i = \{1, \ldots, M\}$. $\overline{F}(t_1, \ldots, t_n)$ is said to be a MNBU survival function. Basic properties of MNBU survival functions are derived. Comparisons and relationships of this new class of MNBU survival functions are developed with earlier classes.

1. Introduction.

The univariate "new better than used" (NBU) class of life distributions was shown by Marshall and Proschan (1972) to play a key role in the study of maintenance policies. See also Barlow and Proschan (1975), Chap. 6.

In this paper, ~~we~~ introduce a multivariate version of the NBU distribution based on a physical model. Shocks occur in time which cause the simultaneous failure of subsets of components. The interval of time until the occurrence of a shock destroying a given subset of components is governed by an NBU distribution. The occurrence times are mutually independent. Note the similarity of this model with the shock model leading to the Marshall-Olkin (1967) multivariate exponential (MVE) distribution. In the Marshall-Olkin model shock times have exponential distributions; in ~~our~~ model, shock times have NBU distributions.

Other versions of the multivariate NBU distributions have been introduced and studied. See, e.g., Marshall and Shaked (1979a, b). Our model may be of interest in certain applications, since the underlying notion derives from a shock model.

In Section 2 we give two equivalent formulations of the MNBU class and obtain its properties. In Section 3, we consider other classes of multivariate new better than used life distributions and compare them with our MNBU class.

2. Definitions and Properties.

In this section the MNBU class is defined and its properties are studied: A fatal shock model generating distributions in this class is formulated. This model is a direct generalization of the Marshall and Olkin (1976) MVE model.

We begin by giving two equivalent definitions of the MNBU distributions:

Definition 2.1. A random vector $\underline{T} \equiv (T_1, \ldots, T_n)$ is said to be a MNBU random vector if it has a representation $T_i = \min_{i \in A} T_A$, where T_A, $A \in I$, are independent NBU random variables (possibly dengerate at 0 or ∞) and I is the class of nonempty subsets of $\{1, \ldots, n\}$.

Definition 2.2. A random vector $\underline{T} \equiv (T_1, \ldots, T_n)$ is said to be a MNBU random vector if it has a representation $T_i = \min_{j \in S_i} X_j$, where $X_1, \ldots, X_m$ are independent NBU random varibles (possibly degenerate at 0 or ∞) and $\phi \neq S_i \subset \{1, \ldots, M\}$, $i = 1, n$, and $\bigcup_{i=1}^{n} S_i = \{1, \ldots, m\}$.

Remark 2.3. The two equivalent formulations above permit us to use whichever is more convenient.

In Definition 2.1, when the random variables T_A, $A \in I$, are exponential, then $\underline{T}$ is the MVE random vector. Recall that $\underline{T}$ can be viewed as the vector of life lengths of n components subject to fatal shocks from independent sources. For every $A \in I$, T_A is the random time at which a shock occurs which simultaneously destroys all the components whose indices form the set A.

Let $\bar{F}(t_1, \ldots, t_n) = P(T_1 > t_1, \ldots, T_n > t_n)$ be the joint survival function of $T_1, \ldots, T_n$, where $\underline{T}$ is MNBU. Eq. (2.1) expresses $\bar{F}(t_1, \ldots, t_n)$

in terms of $\overline{F}$, $A \in I$, where $\overline{F}_A$ is the survival function of T_A:

(2.1) $\overline{F}(t_1, \ldots, t_n) = \prod_{A \in I} \overline{F}_A(\max_{i \in A} t_i)$, $t_i \geq 0$, $i = 1, \ldots, n$.

The following lemma shows that $\overline{F}(t_1, \ldots, t_n)$ enjoys a property similar to the defining property of NBU random variables.

Lemma 2.4. Let $\overline{F}(t_1, \ldots, t_n)$ be defined by (2.1). Then

(2.2) $\overline{F}(t_1 + s, \ldots, t_n + s) \leq \overline{F}(t_1, \ldots, t_n)\overline{F}(s, \ldots, s)$ for all $s \geq 0$, $t_i \geq 0$, $i = 1, \ldots, n$.

Proof. Since $\max_{i \in A}(t_i + s) = \max_{i \in A} t_i + s$, and T_A is NBU for each $A \in I$, we have:

$\overline{F}(t_1 + s, \ldots, t_n + s) = \prod_{A \in I} \overline{F}_A(\max_{i \in A}(t_i + s)) \leq \prod_{A \in I} \overline{F}_A(\max_{i \in A} t_i)\overline{F}_A(s) = \overline{F}(t_1, \ldots, t_n)\overline{F}(s, \ldots, s)$. ||

Remark 2.5. Note that (2.2) can be expressed as $P(T_1 > t_1 + s, \ldots, T_n > t_n + s | T_1 > s, \ldots, T_n > s) \leq P(T_1 > t_1, \ldots, T_n > t_n)$. This asserts that the joint survival probability of n components each of age s is less than or equal to the joint survival probability of n new components. Another alternative interpretation of (2.2) may be obtained by rewriting it as $P(T_1 > t_1 + s, \ldots, T_n > t_n + s | T_1 > t_1, \ldots, T_n > t_n) \leq P(T_1 > s, \ldots, T_n > s)$. This states that a series system of n components of ages $t_1, \ldots, t_n$ is stochastically shorter-lived than is a series system of n new components.

Remark 2.6. A multivariate new worse than used (MNWU) random vector $\underline{T}$ can be defined as in Definition 2.1 (Definition 2.2) where now T_A, $A \in I$, $(X_i, i = 1, \ldots, M)$ are assumed to be independent NWU random variables. If $\overline{F}(t_1, \ldots, t_n)$ denotes the joint survival function of $T_1, \ldots, T_n$, then we can easily show that $\overline{F}(t_1 + s_1, \ldots, t_n + s_n) \geq \overline{F}(t_1, \ldots, t_n)\overline{F}(s_1, \ldots, s_n)$.

Note that in the MNWU case, the s values may differ, while in the MNBU case, the s values must be the same.

The following lemma establishes bounds for the joint distribution and the joint survival function of MNBU random vectors.

<u>Lemma 2.7.</u> Let $\underline{T} \equiv (T_1, \ldots, T_n)$ be MNBU and let $F(t_1, \ldots, t_n)$ and $\overline{F}(t_1, \ldots, t_n)$ be the joint distribution and the joint survival function of $T_1, \ldots, T_n$ respectively. Then

(i) $\overline{F}(t_1, \ldots, t_n) \geq \prod_{i=1}^{n} (\prod_{i \in A} F_A(t_i)),$

(ii) $F(t_1, \ldots, t_n) \geq \prod_{i=1}^{n} [1 - \prod_{i \in A} F_A(t_i)].$

<u>Proof.</u> Since $T_1, \ldots, T_n$ are increasing functions of independent random variables, they are associated. The results in (i) and (ii) follow readily from well known inequalities for associated random variables. ||

The following theorem shows that the MNBU class has many desirable properties.

<u>Theorem 2.8.</u> The following properties hold for the MNBU class:

(P1) Let T be an NBU random variable. Then T is 1-dimensional MNBU.

(P2) Let $T_1, \ldots, T_n$ be independent NBU random variables. Then $\underline{T}$ is MNBU.

(P3) Let $\underline{T}$ be MNBU. Then $(T_{i_1}, \ldots, T_{i_k})$ is k-dimensional MNBU, $1 \leq i_1 < \ldots < i_k \leq n$, $k = 1, \ldots, n$.

(P4) Let $\underline{T}$ be MNBU and $T_j^* = \min_{i \in B_j} T_i$, $\phi \neq B_j \subset \{1, \ldots, n\}$, $j = 1, \ldots, m$. Then $\underline{T}^*$ is MNBU.

(P5) Let $\underline{T}$ be MNBU and $a_i > 0$, $i = 1, \ldots, n$. Then $\min_{1 \leq i \leq n} a_i T_i$ is NBU.

(P6) Let $\underline{T}$ be n-dimensional MNBU, $\underline{T}'$ be m-dimensional MNBU, and $\underline{T}$, $\underline{T}'$ be independent. Then $(\underline{T}, \underline{T}')$ is $(m + n)$ - dimensional MNBU.

(P7) Let $\underline{T}$ be MNBU and let τ be the life function of a coherent system. Then $\tau(\underline{T})$ is NBU.

(P8) Let g: $[0, \infty) \to [0, \infty)$ be a strictly increasing function such that $g(x + y) \leq g(x) + g(y)$ for all x, y. Let $\underline{T}$ be MNBU, then $\underline{T}' \equiv (g(T_1), \ldots, g(T_n))$ is MNBU.

Proof. (P1) and (P2) are obvious.

(P3) and (P4): Since (P3) is a special case of (P4) we need only prove (P4). Let $T_i = \min_{\ell \in S_i} X_\ell$, $i = 1, \ldots, n$. Then $T_j^* = \min_{\ell \in S_j'} X_\ell$, where $S_j' = \bigcup_{i \in B_j} S_i$, $j = 1, \ldots, m$, and thus $\underline{T}^*$ is MNBU.

(P5) Let $T_i = \min_{i \in A} T_A$, $i = 1, \ldots, n$. Then $\min_{m \leq i \leq n} a_i T_i = \min\{(\min_{i \in A} \Lambda_i) T_A\}$, $A \in I$, an NBU random variable, since a series system f independent NBU random variables is NBU.

(P6) The proof is obvious.

(P7) Let $\tau(\underline{T}) = \max_{1 \leq r \leq p} \min_{i \in P_r} T_i$, where $P_1, \ldots, P_r$ are nonempty subsets of $\{1, \ldots, n\}$. But $T_i = \min_{j \in S_i} X_j$, $\phi \neq S_i \subset \{1, \ldots, M\}$, $i = 1, \ldots, n$. Thus $\tau(\underline{T}) \equiv \tau'(\underline{X}) = \max_{1 \leq r \leq p} \min_{i \in \Lambda_r} X_i$, where $\Lambda_r = \bigcup_{i \in P_r} S_i$, $r = 1, \ldots, p$. Since a coherent system of independent NBU components has NBU life length, the desired result follows.

(P8) Let $T_i = \min_{j \in S_i} X_j$, $\phi \neq S_i \subset \{1, \ldots, M\}$. Since g is increasing, we have $g(T_i) = \min_{j \in S_i} g(X_j)$, $i = 1, \ldots, n$. Clearly $g(X_1), \ldots, g(X_m)$ are independent NBU random variables and consequently $g(T_1), \ldots, g(T_n)$ is MNBU. ||

We conclude this section by giving various necessary and sufficient conditions for an MNBU random vector to be MVE.

Theorem 2.9. Let $\underline{T}$ be MNBU. Then the following conditions are equivalent:

(i) $\underline{T}$ is MVE.

(ii) $\min_{1 \le i \le n} a_i T_i$ is exponential for all $a_i > 0$, $i = 1, \ldots, n$.

(iii) $\underline{T}$ has exponential minimums.

(iv) T_i is exponential for $i = 1, \ldots, n$.

(v) $\min_{1 \le i \le n} T_i$ is exponential.

Proof. It suffices to show that (iv) $\Longrightarrow$ (ii) and (v) $\Longrightarrow$ (i). We only prove that (v) $\Longrightarrow$ (i) since the proof of (iv) $\Longrightarrow$ (ii) is similar. Let $T_i = \min_{j \in S_i} X_j$, $\phi \neq S_i \subset \{1, \ldots, M\}$, where $X_1, \ldots, X_M$ are independent NBU random variables. Now $\min_{1 \le i \le n} T_i = \min_{1 \le j \le m} X_j$, which is exponential. Consequently, each X_j is exponential, and so $\underline{T}$ is MVE. ||

3. Other Classes of Multvariate New Better than Used Distributions and Their Relation to the MNBU Class.

Several alternative definitions are available of multivariate life distributions extending the univariate concept of NBU. Each of these classes satisfies some of the properties which one would expect for a class of multivariate new better than used distributions. In this section we compare the MNBU class with some of these other classes.

Consider nonnegative random variables $T_1, \ldots, T_n$ whose joint distribution satisfies on of the following conditions:

(A) $T_1, \ldots, T_n$ are independent and each T_i is an NBU random variable.

(B) $(T_1, \ldots, T_n)$ is MNBU.

(C) For all $a_i > 0$, $i = 1, \ldots, n$, $\min_{1 \le i \le n} a_i T_i$ is NBU.

(D) For each $\phi \neq A \subset \{1, \ldots, n\}$, $\min_{i \in A} T_i$ is an NBU random variable.

(E) Each T_i is an NBU random variable.

Each of the classes of multivariate distributions defined by (A) - (E) may be designated as a class of multivariate new better than used distributions. We now compare these classes. Clearly (A) $\Longrightarrow$ (B) $\Longrightarrow$ (C) $\Longrightarrow$ (D) $\Longrightarrow$ (E). The following examples (see Esary and Marshall, 1974) show that no other implication among the above classes is possible.

<u>Example 3.1.</u> Let $T_1 = \min(U, W)$, $T_2 = \min(V, W)$, where U, V, W are independent exponential random varibles with parameters $\lambda_1 = \lambda_2 = \lambda_{12} = 1$. Then (T_1, T_2) is MNBU, but T_1, T_2 are not independent. Thus (B) $\not\Longrightarrow$ (A).

<u>Example 3.2.</u> Let $T_1' = 2T_1$, $T_2' = T_2$, where T_1, T_2 are defined in Example 3.1. Obviously $\min(a_1 T_1', a_2 T_2')$ is NBU for all $a_1 > 0$, $a_2 > 0$. However (T_1', T_2') is not MVE. By Theorem 2.7, (T_1', T_2') is not MNBU. Thus (C) $\not\Longrightarrow$ (B).

<u>Example 3.3.</u> Let T_1, T_2 be as in Example 3.1 and let $(T_1^*, T_2^*) = (\min(U, W), \frac{1}{2}W)$. Let $\overline{F}(t_1, t_2) = p\overline{F}_{T_1, T_2}(t_1, t_2) + (1 - p)\overline{F}_{T_1^*, T_2^*}(t_1, t_2)$, where $0 < p < 1$. Let (T_1', T_2') be the bivariate random vector whose joint survival function is $\overline{F}(t_1, t_2)$. Obviously T_1', T_2', and $\min(T_1', T_2')$ are exponential, but $\min(\frac{1}{2}T_1', T_2')$ is not NBU. To see this, let $\overline{F}(t) \equiv P(\min(\frac{1}{2}T_1', T_2') > t) = pe^{-5t} + (1 - p)e^{-4t}$. It is easy to verify that $\overline{F}(2t) > [\overline{F}(t)]^2$ for sufficiently large t. Thus (D) $\not\Longrightarrow$ (C).

<u>Example 3.4.</u> Let U, V, and W be as in Example 3.1. Let $\overline{F}(t_1, t_2) = p\overline{F}_{U,V}(t_1, t_2) + (1 - p)\overline{F}_{W,W}(t_1, t_2)$, where $0 < p < 1$. It is easy to verify that $\overline{F}(t_1, t_2)$ is the joint survival function of a bivariate random vector whose marginals are NBU but whose minimum is not NBU. Thus (E) $\not\Longrightarrow$ (D).

Esary and Marshall (1974) show that if $\underline{T}$ has exponential minimums, then there exists an MVE random vector $\underline{T}'$ such that $\tau(\underline{T})$ and $\tau(\underline{T}')$ are identically distributed for all coherent life functions τ. Unfortunately the class (D) above does not enjoy this property, as is illustrated by the following example.

<u>Example 3.5.</u> Let T_1, T_2 be independent exponential random variables with parameters $\lambda_1 > 0$, $\lambda_2 > 0$, respectively. Then $(T_1 \vee T_2, T_2)$ has NBU minimums. Now assume there exists $(T_1', T_2') \equiv (\min(U, W), \min(V, W))$ such that $\tau(\underline{T})$ and $\tau(\underline{T}')$ are identically distributed, where U, V, and W are independent NBU random variables. We then have $\overline{F} \circ \overline{G} \circ \overline{H} = \overline{F}_2$ and $\overline{F} \cdot \overline{H} = \overline{F}_1$, where $\overline{F}_1$, $\overline{F}_2$, $\overline{F}$, $\overline{G}$, and $\overline{H}$ are the survival functions of $T_1 \vee T_2$, T_2, U, V, and W respectively. This leads to the conclusion that $\overline{G} = \overline{F}_2/\overline{F}_1$, which is impossible.

Finally, we present two additional classes of multivariate new better than used distributions and compare them with the MNBU class. The first of these two classes is due to Marshall and Shaked (1979a), the second is essentially due to Block and Savits (1979).

<u>Definition (F).</u> A random vector $\underline{T}$ is said to be <u>multivariate new better than used (F)</u> if $P\{T_i \geq 0, i = 1, \ldots, n\} = 1$ and $P\{\underline{T} \in (\alpha + \beta)A\} \leq P\{\underline{T} \in \alpha A\} P\{\underline{T} \in \beta A\}$ for every $\alpha \geq 0$, $\beta \geq 0$, and every open upper set $A \subset [0, \infty)^n$.

<u>Definition (G).</u> A random $\underline{T}$ is said to be <u>multivariate new better than used (G)</u> if $\underline{T}$ has a representation $T_i = \sum_{j \in S_i} X_j$, where $X_1, \ldots, X_m$ are independent NWU and $\emptyset \neq S_i \subset \{1, \ldots, M\}$, $i = 1, \ldots, n$.

The following lemma shows that the MNBU class contained in MNBU (F).

Lemma 3.6. Let $\underline{T}$ be MNBU. Then $\underline{T}$ satisfies the conditions of Definition (F).

Proof. Obviously $P(T_i \geq 0, i = 1, \ldots, n) = 1$. Now $T_i = \min_{j \in S_i} X_j$, where $X_1, \ldots, X_n$ are independent NBU random variables, and $\phi \neq S_i \subset \{1, \ldots, N\}$, $i = 1, \ldots, n$. The desired result follows immediately by Property 3.4 of Marshall and Shaked (1979a). ||

The following examples show that no other implication holds between our MNBU class and the MNBU (F) or the MNBU (G) classes.

Example 3.7. Let $\overline{F}(x, y) = e^{-\sqrt{x^2+y^2}}$ $x \geq 0$, $y \geq 0$. It can be shown that the bivariate random vector (X, Y) whose joint survival function is $\overline{F}(x, y)$ satisfies (F). Theorem 2.7 shows that (X, Y) cannot be MNBU.

Example 3.8. Let U, V, and W be independent exponential random variables with parameters $\lambda_1 \neq \lambda_2$ and $\lambda_{12} > 0$ respectively. Let $T_1 = \min(U, W)$ and $T_2 = \min(V, W)$. Clearly (T_1, T_2) is MNBU, but it is not MNBU (G). To see this, assume $T_1 = X + Z$, $T_2 = Y + Z$, where X, Y, and Z are independent NBU random variables. Since T_1 is exponential, it follows that either X is exponential and Z degenerate at 0, or vice versa; similarly for Y and Z. Consequently, T_1 and T_2 are either independent or identically distributed, which is impossible.

Example 3.9. Let X, Y, and Z be independent with absolutely continuous distributions. Let $T_1 = X + Z$, $T_2 = Y + Z$. Then (T_1, T_2) is MNBU (G), but cannot be MNBU. For if (T_1, T_2) were MNBU, then T_1 and T_2 would be independent, which is not the case.

Remark 3.10. In Example 3.9, observe that $(T_1, T_2) = (X, Y) + (Z, Z)$. This shows that the MNBU class is not closed under convolution.

REFERENCES.

Barlow, R. E. and Proschan, F. (1975). Statistical Theory of Reliability and Life Testing: Probability Models. Holt, Rinehart and Winston, New York.

Block, H. W. and Savits, T. H. (1979). Multivariate IFRA distributions. Ann. Probability, to appear.

Esary, J. D. and Marshall, A. W. (1974). Multivariate distributions with exponential minimums. Ann. Statistics, 2 84-96.

Esary, J. D. and Marshall, A. W. (1979). Multivariate distributions with increasing hazard rate average. Ann. Probability, 7 359-370.

Marshall, A. W. and Olkin, I. (1967). A multivariate exponential distribution. J. Amer. Statist. Assoc., 62 30-44.

Marshall, A. W. and Proschan, F. (1972). Classes of distributions applicable in replacement, with renewal theory implications, Proc. 6th Berkeley Symp. Math. Stat. Prob., I, ed. by L. LeCam, J. Neyman, and E. L. Scott, 395-415. Univ. of California Press, Bekeley, Calif.

Marshall, A. W. and Shaked, M. (1979a). A class of multivariate new better than used distributions. Unpublished report.

Marshall, A. W. and Shaked, M. (1979b). Multivariate distributions with new better than used marginals. In preparation.

UNCLASSIFIED

Security Classification of this Page

REPORT DOCUMENTATION PAGE

1. REPORT NUMBERS
FSU No. M540
AFOSR No. 78-105
USARO No. D45

2. GOVT. ACCESSION NO.
AD-A084 794

3. RECIPIENT'S CATALOG NUMBER

4. TITLE
A Multivariate New Better Than Used Class Derived from a Shock Model.

5. TYPE OF REPORT & PERIOD COVERED
Technical Report,

6. PERFORMING ORG. REPORT NUMBER
FSU No. M540

7. AUTHOR(s)
Emad El-Neweihi
Frank Proschan
Jayaram Sethuraman

8. CONTRACT OR GRANT NUMBER(s)
~~AFOSR No. 78-105~~
~~USARO No. D45~~
DAAG29-79-C-0158

9. PERFORMING ORGANIZATION NAME & ADDRESS
The Florida State University
Department of Statistics
Tallahassee, Florida 32306

10. ~~PROGRAM ELEMENT~~, PROJECT ~~TASK AREA AND WORK~~ UNIT NOS.
AFOSR

11. CONTROLLING OFFICE NAME & ADDRESS
U.S. Army Research Office - Durham
P.O. Box 12211
Research Triangle Park, North Carolina 27709

The U.S. Air Force
Air Force Office of Scientific Research
Bolling Air Force Base, D.C. 20332

12. REPORT DATE
March 1980

13. NUMBER OF PAGES
10

14. MONITORING AGENCY NAME & ADDRESS
(if different from Controlling Office)

15. SECURITY CLASS (of this report)
Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)
Approved for public release: distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS
New better than used, multivariate new better than used, shock model, multivariate exponential, reliability, life distribution, survival function.

400277

20. ABSTRACT (CON'T)

We introduce a new class of multivariate new better than used (MNBU) life distributions based on a shock model similar to that yielding the Marshall-Olkin multivariate exponential distribution. Let $T_1, \ldots, T_N$ be independent new better than used (NBU) life lengths. Let $\overline{F}(t_1, \ldots, t_n)$ be the joint survival function of $\min_{j \in A_i} T_j$, $i = 1, \ldots, n$, where $A_1, \ldots, A_n$ are nonempty subsets of $\{1, \ldots, N\}$ and $\bigcup_{i=1}^{n} A_i = \{1, \ldots, N\}$. $\overline{F}(t_1, \ldots, t_n)$ is said to be a MNBU survival function. Basic properties of MNBU survival functions are derived. Comparisons and relationships of this new class of MNBU survival functions are developed with earlier classes.